Name: _____ **100 points total**

CS 2123 Homework 1 Fall 2019

Assignment is due at 11:59pm on September 19. Submit a digital copy of the assignment on Harvey. Submit the file as a single PDF named HW1LastNameFirstName.pdf. If you are working with a partner, turn in one assignment.

You may submit a lateness coupon request BEFORE the assignment is due by sending an email to cs2123f19@googlegroups.com with Subject "CS2123 Lateness Coupon". All other late work will receive a 10 percentage point deduction per day (including weekends), No late work is accepted beyond five days after the assignment is due.

Q1 (5)	_____
Q2 (5)	_____
Q3 (18)	_____
Q4 (12)	_____
Q5 (12)	_____
Q6 (12)	_____
Q7 (10)	_____
Q8 (8)	_____
Q9 (5)	_____
Q10 (12)	_____
Q11 (1)	_____
Total (100)	_____

Q1. (5 points) Decide whether the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

> In every instance of the Stable Matching Problem, there is a stable matching containing a pair (m, w) such that m is ranked first on the preference list of w and w is ranked first on the preference list of m.

Q2. (5 points) Decide whether the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

> Consider an instance of the Stable Matching Problem in which there exists a man m and woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m. Then in every stable matching S for this instance, the pair (m, w) belongs to S.

Q3. (18 points)

a. Show that $3n^2 + 4n$ is $O(n^2)$ by applying the definition of Big-Oh.

b. Show that $n \log n + 17$ is $\Omega(n)$ by applying the definition of Big-Omega.

b. Show that $n^2 + 4n$ is $\Theta(n^2)$ by applying the definition of Big-Theta.

Q4. (12 points)

a. Does $2n^2$ dominate $4n^2 + 3n$? Show why or why not.

b. Does n^3 dominate $6n^2$? Show why or why not.

Q5. (12 points)
State whether the following is True or False, along with a brief explanation of your reasoning.

a. $2^{n+2} = O(2^n)$?

b. $n \cdot 2^n = O(2^n)$?

Q6. (12 points)

For each of the following pairs of functions, either $f(n)$ is in $O(g(n))$, $f(n)$ is in $\Omega(g(n))$, or $f(n) = \Theta(g(n))$ (i.e., $f(n)$ is in both $O(g(n))$ and $\Omega(g(n))$). Determine which relationship is correct, no explanation required.

a. $f(n) = \log_2(2n)$; $g(n) = \log_2(n) + 2$

b. $f(n) = n^2 + 3n + 4$, $g(n) = 6n + 7$

c. $f(n) = \log 100$; $g(n) = 5$

d. $f(n) = \log n + n$; $g(n) = n \log n$

e. $f(n) = 2^n$; $g(n) = n^2$

f. $f(n) = 2^n - n^2$, $g(n) = n^4 + n^2$

Q7. (10 points) True or False? (No explanation needed)

a. $2n^2 + 1 = \Omega(n^2)$

b. $\log n = o(\sqrt{n})$

c. $\sqrt{n} = o(\log n)$

d. $\sqrt{n}\log(n) = O(n \log n)$

e. $n \log(n) = O(\sqrt{n} \log n)$

Q8. (8 points)
For each of these questions, briefly explain your answer.

a. Explain why saying "The running time for my algorithm is at least $O(n^2)$" is meaningless.

b. If I prove that an algorithm takes $\Theta(n^2)$ worst-case time, is it possible that it takes $\Theta(n^3)$ on some inputs?

c. If I prove that an algorithm takes $\Theta(n^2)$ worst-case time, is it possible that it takes $\Theta(n)$ on some inputs?

d. If I prove that an algorithm takes $\Theta(n^2)$ worst-case time, is it possible that it takes $\Theta(n)$ on all inputs?

Q9. (5 points)

Take the following list of functions and arrange them in ascending order of growth rate. That is, if function $g(n)$ immediately follows function $f(n)$ in your list, then it should be the case that $f(n)$ is $O(g(n))$.

$$f_1(n) = n^{2.5}$$
$$f_2(n) = \sqrt{2n}$$
$$f_3(n) = n + 10$$
$$f_4(n) = 10^n$$
$$f_5(n) = 100^n$$
$$f_6(n) = n^2 \log n$$

Q10. (12 points)

Assume you have functions f and g such that $f(n)$ is $O(g(n))$. Decide whether each of the following statements is true or false and give a proof or counterexample.

a. $2^{f(n)}$ is $O(2^{g(n)})$.

b. $(f(n))^2$ is $O((g(n))^2)$.

Q11. (1 point) How long (in hours) did you spend on this assignment? (full credit for any truthful answer)